B.B.S.S. Sec School
CLASS-IX
SUB: MATHS
Ch-2: Polynomials (Part II)

(1)

Remainder Theorem:- Let $p(x)$ be a polynomial of degree greater than 1 and α be any real number. If $p(x)$ is divided by $(x-\alpha)$ then the remainder is $p(\alpha)$

Working Rules:- To find the remainder when $p(x)$ is divisible by $q(x)$, where $q(x) = x+a$ is a linear polynomial

Step 1: Put $q(x) = 0$, then $x+a=0$ or $x=-a$

Step 2: Put $x=-a$ in $p(x)$ to get $p(-a)$

Step 3: Find the value of $p(-a)$, which will ~~give you~~ the required remainder.

Factor Theorem:-

Let $p(x)$ be a polynomial of degree ≥ 1 and a is any real number. Then

(I) $p(a)=0 \Rightarrow (x-a)$ is a factor of $p(x)$.

(II) $(x-a)$ is a factor of $p(x) \Rightarrow p(a)=0$.

Working rules:- To verify $g(x) = x+a$ as a factor of polynomial $f(x)$.

Step 1: obtain $g(x) = x+a$.

Step 2: Put $x=-a$ in $f(x)$ to get $f(-a)$.

Step 3: If $f(-a)$ becomes zero then $g(x)$ is a factor of $f(x)$, otherwise not.

P.T.O.

NCERT Ex-2.3

Q.1 (i) Here $p(x) = x^3 + 3x^2 + 3x + 1$

By remainder theorem, the required remainder is equal to $p(-1)$

$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$p(-1) = -1 + 3\times1 - 3 + 1$

$p(-1) = -1 + 3 - 3 + 1$

$p(-1) = 4 - 4 = 0$ Ans

(iv) Here $p(x) = x^3 + 3x^2 + 3x + 1$

By remainder theorem the required remainder is equal to $p(-\pi)$

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3\times-\pi + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$ Ans

Factorization by splitting of middle term:-

Steps:- (i) Write the quadratic polynomial in standard form $ax^2 + bx + c$

(ii) Find out the numbers p and q such that $p+q = b$ and $pq = ac$.

(iii) Write the middle term i.e. bx as $px + q$.

(iv) factorise by grouping the terms

$ax^2 + bx + c = ax^2 + px + qx + c$

Example $3x^2 - 8x + 5$

$a=3, b=-8, c=5$

$a\times c = 5\times3 = 15$

Find out two factors of 15 such that their sum is -8

$-5\times-3 = 15$, $(-5) + (-3) = 8$

$\therefore 3x^2 - 8x + 5 = 3x^2 - 5x - 3x + 5 \Rightarrow x(3x-5) - 1(3x-5)$
$= (x-1)(3x-5)$ Ans

Ex 2.4 NCERT

Q1 (i) Here $p(x) = x^3 + x^2 + x + 1$

$x + 1 = 0 \Rightarrow x = -1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 2 - 2$

$= 0$

$\therefore$ By factor theorem $(x+1)$ is a factor of $p(x)$

Q.3 (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$g(x) = x - 1 \therefore x = 1$

$p(1) = 2 \times 1^2 + k \times 1 + \sqrt{2}$

$p(1) = 2 + \sqrt{2} + k = 0$

$\therefore k = -(2 + \sqrt{2})$

Q.4 (ii) $12x^2 - 7x + 1$

$\Rightarrow 12x^2 - 4x - 3x + 1$ [By spilting the middle term]

$4x(3x-1) - 1(3x-1)$

$\Rightarrow (3x-1)(4x-1)$ Ans

Q.5 (ii) $p(x) = x^3 - 3x^2 - 9x - 5$

We shall look all factor of -5

$\pm 1, \pm 5$

First we traile $p(5)$

$p(5) = (5)^3 - 3 \times 5^2 - 9 \times 5 - 5$

$= 125 - 75 - 45 - 5$

$= 125 - 125$

$= 0$

$\therefore (x-5)$ is a factor of $p(x)$

Now we divided $p(x)$ by $x - 5$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x-5 & x^3 - 3x^2 - 9x - 5 \\
 & -x^3 \mp 5x^2 \\
\hline
 & 2x^2 - 9x \\
 & -2x^2 \mp 10x \\
\hline
 & +x - 5 \\
 & -x \mp 5 \\
\hline
 & 0
\end{array}
$$

$\Rightarrow x^3 - 3x^2 - 9x - 5 = (x-5)(x^2 + 2x + 1)$

$\Rightarrow (x-5)[x^2 + x + x + 1]$

$(x-5)[x(x+1) + 1(x+1)]$

$(x-5)(x+1)(x+1)$

Ans

Some Important Identities! -

(i) $(x+y)^2 = x^2+2xy+y^2$

(ii) $(x-y)^2 = x^2-2xy+y^2$

(iii) $x^2-y^2 = (x+y)(x-y)$

(iv) $(x+a)(x+b) = x^2+(a+b)x+ab$.

(v) $(x+y)^3 = x^3+y^3+3xy(x+y)$

(vi) $(x-y)^3 = x^3-y^3-3xy(x-y)$

(vii) $x^3+y^3 = (x+y)(x^2-xy+y^2)$

(viii) $x^3-y^3 = (x-y)(x^2+xy+y^2)$

(ix) $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

(x) If $x+y+z=0$ then $x^3+y^3+z^3 = 3xyz$

NCERT
Ex 2.5 Q1 (i) $(x+4)(x+10)$

$\because (x+a)(x+b) = x^2+(a+b)x+ab$

$\therefore (x+4)(x+10) = x^2+(10+4)x+10\times4$

$= x^2+14x+40$ Ans

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$= (y^2)^2-\left(\frac{3}{2}\right)^2$

$= y^4-\frac{9}{4}$ Ans

Q.3

(i) $9x^2+6xy+y^2$

$(3x)^2+2(3x)(y)+(y)^2$ $[a^2+2ab+b^2 = (a+b)^2]$

$(3x+y)^2 = (3x+y)(3x+y)$ Ans

Q.4 (ii) $(2x-y+z)^2$ $[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac]$

$(2x-y+z)^2 = (2x)^2+(-y)^2+(z)^2+2(2x)(-y)(z)+2(-y)(z)+2(2x)(z)$

$(2x-y+z)^2 = 4x^2+y^2+z^2 - 4xyz - 2yz + 4xz$

Q.6 $(2x+1)^3 = (2x)^3+(1)^3+3(2x)(1)(2x+1)$

$= 8x^3+1+6x(2x+1)$

$= 8x^3+1+12x^2+6x$ Ans

Q.7 (i) $99^3 = (100-1)^3$

$\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$\therefore 99^3 = (100-1)^3 = 100^3 - 1^3 - 3\times100\times1(100-1)$

$= 1000000 - 1 - 300 \times 99$

$= 1000000 - 1 - 29700$

$= 1000000 - 29701$

$= 970299$ Ans

Q.8 (v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$ $[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3]$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$ Ans

Q.10 (ii) $27y^3 + 125z^3$ $[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (3y)^3 + (5z)^3 = (3y+5z)(9y^2 - 15yz + 25z^2)$

Q.13 $\because x + y + z = 0$

$\Rightarrow x + y = -z$ — (1)

on cubing both sides

$(x+y)^3 = (-z)^3$

$x^3 + y^3 + 3xy(x+y) = -z^3$

$x^3 + y^3 + 3xy \times -z = -z^3$

$x^3 + y^3 - 3xyz = -z^3$

$x^3 + y^3 + z^3 = 3xyz$ HP

(14) (i) $(-12)^3 + 7^3 + 5^3$

$a + b + c = -12 + 7 + 5$

$= -12 + 12$

$a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$(-12)^3 + (7)^3 + (5)^3 = 3\times -12 \times 7 \times 5$

$= -1260$ Ans

(15) Area $= 25a^2 - 35a + 12$

$= 25a^2 - 15a - 20a + 12$

$\Rightarrow 5a(5a-3) - 4(5a-3)$

$\Rightarrow (5a-4)(5a-3)$

Ans

HOMEWORK:-

NCERT

Ex 2.4: Q.1 (ii) to (iv)
Q.2 (ii) (iii)
Q.3 (i) (iii) (iv)
Q.4 (ii) to (iv)
Q.5 (i) (iii) (iv)

Ex 2.5

Q.1 (ii) to (v)
Q.2.
Q.3 (ii) (iii)
Q.4 (ii) to (VI)
Q.5 to Q.8
Q.9 (ii)
Q.10 (ii)
Q.11, 12
Q.14 (ii)
Q.15, 16

ASSIGNMENT:

(I) Factorize

(i) $x^2+7x+12$

(ii) $2x^2-5x+3$

(iii) $3x^2-5x+2$

(iv) $6x^2+7x-3$

(v) $2x^3-3x^2-17x+30$

(vi) x^3+x^2-4x-4

(II) Evaluate (with the help of identities)

(i) $(104)^3$ (ii) $(98)^3$

(iii) 27^3-17^3 (iv) 106^3-94^3